

Characteristics of a fluid

2.1 Fluid

Fluids are divided into liquids and gases. A liquid is hard to compress and as in the ancient saying ‘Water takes the shape of the vessel containing it’, it changes its shape according to the shape of its container with an upper free surface. Gas on the other hand is easy to compress, and fully expands to fill its container. There is thus no free surface.

Consequently, an important characteristic of a fluid from the viewpoint of fluid mechanics is its compressibility. Another characteristic is its viscosity. Whereas a solid shows its elasticity in tension, compression or shearing stress, a fluid does so only for compression. In other words, a fluid increases its pressure against compression, trying to retain its original volume. This characteristic is called compressibility. Furthermore, a fluid shows resistance whenever two layers slide over each other. This characteristic is called viscosity.

In general, liquids are called incompressible fluids and gases compressible fluids. Nevertheless, for liquids, compressibility must be taken into account whenever they are highly pressurised, and for gases compressibility may be disregarded whenever the change in pressure is small. Although a fluid is an aggregate of molecules in constant motion, the mean free path of these molecules is $0.06\ \mu\text{m}$ or so even for air of normal temperature and pressure, so a fluid is treated as a continuous isotropic substance.

Meanwhile, a non-existent, assumed fluid without either viscosity or compressibility is called an ideal fluid or perfect fluid. A fluid with compressibility but without viscosity is occasionally discriminated and called a perfect fluid, too. Furthermore, a gas subject to Boyle’s–Charles’ law is called a perfect or ideal gas.

2.2 Units and dimensions

All physical quantities are given by a few fundamental quantities or their combinations. The units of such fundamental quantities are called base

units, combinations of them being called derived units. The system in which length, mass and time are adopted as the basic quantities, and from which the units of other quantities are derived, is called the absolute system of units.

2.2.1 Absolute system of units

MKS system of units

This is the system of units where the metre (m) is used for the unit of length, kilogram (kg) for the unit of mass, and second (s) for the unit of time as the base units.

CGS system of units

This is the system of units where the centimetre (cm) is used for length, gram (g) for mass, and second (s) for time as the base units.

International system of units (SI)

SI, the abbreviation of La Syst eme International d'Unit es, is the system developed from the MKS system of units. It is a consistent and reasonable system of units which makes it a rule to adopt only one unit for each of the various quantities used in such fields as science, education and industry.

There are seven fundamental SI units, namely: metre (m) for length, kilogram (kg) for mass, second (s) for time, ampere (A) for electric current, kelvin (K) for thermodynamic temperature, mole (mol) for mass quantity and candela (cd) for intensity of light. Derived units consist of these units.

Table 2.1 Dimensions and units

Quantity	Absolute system of units			Units
	α	β	γ	
Length	1	0	0	m
Mass	0	1	0	kg
Time	0	0	1	s
Velocity	1	0	-1	m/s
Acceleration	1	0	-2	m/s ²
Density	-3	1	0	kg/m ³
Force	1	1	-2	N = kg m/s ²
Pressure, stress	-1	1	-2	Pa = N/m ²
Energy, work	2	1	-2	J
Viscosity	-1	1	-1	Pa s
Kinematic viscosity	2	0	-1	m ² /s

2.2.2 Dimension

All physical quantities are expressed in combinations of base units. The index number of the combination of base units expressing a certain physical quantity is called the dimension, as follows.

In the absolute system of units the length, mass and time are respectively expressed by L, M and T. Put Q as a certain physical quantity and c as a proportional constant, and assume that they are expressed as follows:

$$Q = cL^\alpha M^\beta T^\gamma \quad (\text{SI}) \quad (2.1)$$

where α , β and γ are respectively called the dimensions of Q for L, M, T. Table 2.1 shows the dimensions of various quantities.

2.3 Density, specific gravity and specific volume

The mass per unit volume of material is called the density, which is generally expressed by the symbol ρ . The density of a gas changes according to the pressure, but that of a liquid may be considered unchangeable in general. The units of density are kg/m^3 (SI). The density of water at 4°C and 1 atm (101 325 Pa, standard atmospheric pressure; see Section 3.1.1) is 1000 kg/m^3 .

The ratio of the density of a material ρ to the density of water ρ_w is called the specific gravity, which is expressed by the symbol s :

$$s = \rho/\rho_w \quad (2.2)$$

The reciprocal of density, i.e. the volume per unit mass, is called the specific volume, which is generally expressed by the symbol v :

$$v = 1/\rho \quad (\text{m}^3/\text{kg}) \quad (2.3)$$

Values for the density ρ of water and air under standard atmospheric pressure are given in Table 2.2.

Table 2.2 Density of water and air (standard atmospheric pressure)

Temperature ($^\circ\text{C}$)		0	10	15	20	40	60	80	100
ρ (kg/m^3)	Water	999.8	999.7	999.1	998.2	992.2	983.2	971.8	958.4
	Air	1.293	1.247	1.226	1.205	1.128	1.060	1.000	0.9464

2.4 Viscosity

As shown in Fig. 2.1, suppose that liquid fills the space between two parallel plates of area A each and gap h , the lower plate is fixed, and force F is needed to move the upper plate in parallel at velocity U . Whenever $Uh/\nu < 1500$ ($\nu = \mu/\rho$: kinematic viscosity), laminar flow (see Section 4.4) is maintained, and a linear velocity distribution, as shown in the figure, is obtained. Such a parallel flow of uniform velocity gradient is called the Couette flow.

In this case, the force per unit area necessary for moving the plate, i.e. the shearing stress (Pa), is proportional to U and inversely proportional to h . Using a proportional constant μ , it can be expressed as follows:

$$\tau = \frac{F}{A} = \mu \frac{U}{h} \quad (2.4)$$

The proportional constant μ is called the viscosity, the coefficient of viscosity or the dynamic viscosity.

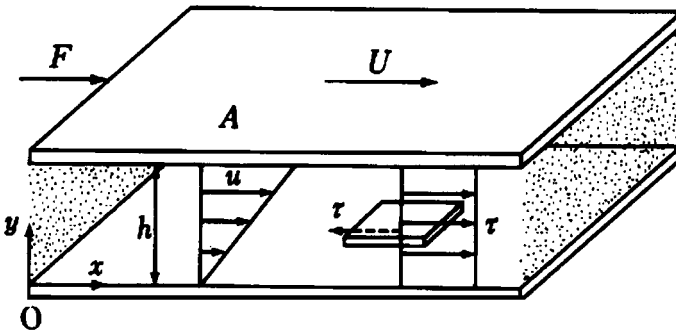


Fig. 2.1 Couette flow

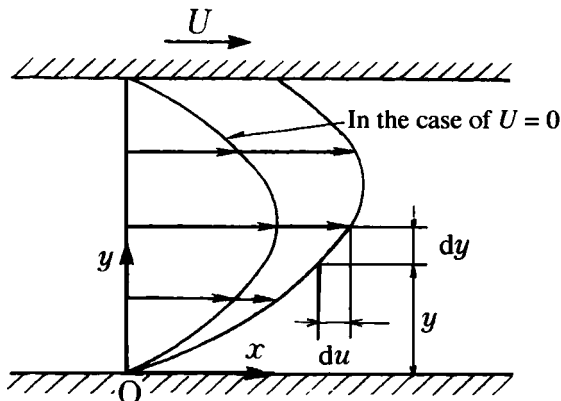


Fig. 2.2 Flow between parallel plates

Isaac Newton (1642–1727)

English mathematician, physicist and astronomer; studied at the University of Cambridge. His three big discoveries of the spectral analysis of light, universal gravitation and differential and integral calculus are only too well known. There are so many scientific terms named after Newton (Newton's rings and Newton's law of motion/viscosity/resistance) that he can be regarded as the greatest contributor to the establishment of modern natural science.



Newton's statue at Grantham near Woolsthorpe, his birthplace

Such a flow where the velocity u in the x direction changes in the y direction is called shear flow. Figure 2.1 shows the case where the fluid in the gap is not flowing. However, the velocity distribution in the case where the fluid is flowing is as shown in Fig. 2.2. Extending eqn (2.4) to such a flow, the shear stress τ on the section dy , distance y from the solid wall, is given by the following equation:

$$\tau = \mu \frac{du}{dy} \quad (2.5)$$

This relation was found by Newton through experiment, and is called Newton's law of viscosity.

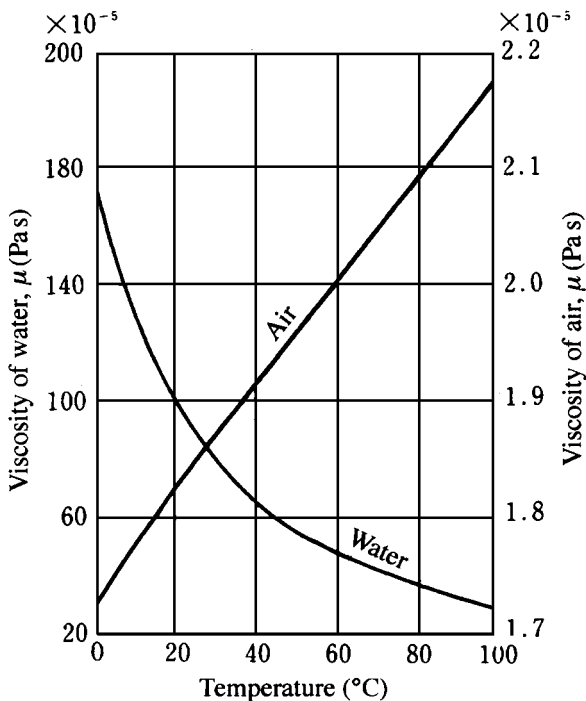


Fig. 2.3 Change in viscosity of air and of water under 1 atm

In the case of gases, increased temperature makes the molecular movement more vigorous and increases molecular mixing so that the viscosity increases. In the case of a liquid, as its temperature increases molecules separate from each other, decreasing the attraction between them, and so the viscosity decreases. The relation between the temperature and the viscosity is thus reversed for gas and for liquid. Figure 2.3 shows the change with temperature of the viscosity of air and of water.

The units of viscosity are Pa s (pascal second) in SI, and g/(cm s) in the CGS absolute system of units. 1g/(cm s) in the absolute system of units is called 1 P (poise) (since Poiseuille's law, stated in Section 6.3.2, is utilised for measuring the viscosity, the unit is named after him), while its 1/100th part is 1 cP (centipoise). Thus

$$1 \text{ P} = 100 \text{ cP} = 0.1 \text{ Pa s}$$

The value ν obtained by dividing viscosity μ by density ρ is called the kinematic viscosity or the coefficient of kinematic viscosity:

$$\nu = \frac{\mu}{\rho} \quad (2.6)$$

Since the effect of viscosity on the movement of fluid is expressed by ν , the name of kinematic viscosity is given. The unit is m^2/s regardless of the system of units. In the CGS system of units $1 \text{ cm}^2/\text{s}$ is called 1 St (stokes) (since

Table 2.3 Viscosity and kinematic viscosity of water and air at standard atmospheric pressure

Temp. (°C)	Water		Air	
	Viscosity, μ (Pa s $\times 10^5$)	Kinematic viscosity, ν (m ² /s $\times 10^6$)	Viscosity, μ (Pa s $\times 10^5$)	Kinematic viscosity, ν (m ² /s $\times 10^6$)
0	179.2	1.792	1.724	13.33
10	130.7	1.307	1.773	14.21
20	100.2	1.004	1.822	15.12
30	79.7	0.801	1.869	16.04
40	65.3	0.658	1.915	16.98

Stokes' equation, to be stated in Section 9.3.3, is utilised for measuring the viscosity, it is named after him), while its 1/100th part is 1 cSt (centistokes). Thus

$$1 \text{ St} = 1 \times 10^{-4} \text{ m}^2/\text{s}$$

$$1 \text{ cSt} = 1 \times 10^{-6} \text{ m}^2/\text{s}$$

The viscosity μ and the kinematic viscosity ν of water and air under standard atmospheric pressure are given in Table 2.3.

The kinematic viscosity ν of oil is approximately 30–100 cSt. Viscosity

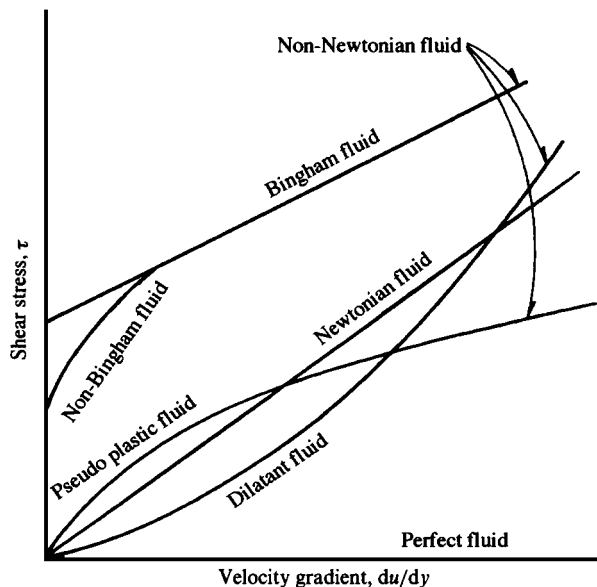


Fig. 2.4 Rheological diagram

sensitivity to temperature is expressed by the viscosity index VI,¹ a non-dimensional number. A VI of 100 is assigned to the least temperature sensitive oil and 0 to the most sensitive. With additives, the VI can exceed 100. While oil is used under high pressure in many cases, the viscosity of oil is apt to increase somewhat as the pressure increases.

For water, oil or air, the shearing stress τ is proportional to the velocity gradient du/dy . Such fluids are called Newtonian fluids. On the other hand, liquid which is not subject to Newton's law of viscosity, such as a liquid pulp, a high-molecular-weight solution or asphalt, is called a non-Newtonian fluid. These fluids are further classified as shown in Fig. 2.4 by the relationship between the shearing stress and the velocity gradient, i.e. a rheological diagram. Their mechanical behaviour is minutely treated by rheology, the science allied to the deformation and flow of a substance.

2.5 Surface tension

The surface of a liquid is apt to shrink, and its free surface is in such a state where each section pulls another as if an elastic film is being stretched. The tensile strength per unit length of assumed section on the free surface is called the surface tension. Surface tensions of various kinds of liquid are given in Table 2.4.

As shown in Fig. 2.5, a dewdrop appearing on a plant leaf is spherical in shape. This is also because of the tendency to shrink due to surface tension. Consequently its internal pressure is higher than its peripheral pressure. Putting d as the diameter of the liquid drop, T as the surface tension, and p as the increase in internal pressure, the following equation is obtained owing to the balance of forces as shown in Fig. 2.6:

$$\pi d T = \frac{\pi d^2}{4} \Delta p$$

or

$$\Delta p = 4T/d \quad (2.7)$$

The same applies to the case of small bubbles in a liquid.

Table 2.4 Surface tension of liquid (20°C)

Liquid	Surface liquid	N/m
Water	Air	0.0728
Mercury	Air	0.476
Mercury	Water	0.373
Methyl alcohol	Air	0.023

¹ ISO 2909-1981



Fig. 2.5 A dewdrop on a taro leaf

Whenever a fine tube is pushed through the free surface of a liquid, the liquid rises up or falls in the tube as shown in Fig. 2.7 owing to the relation between the surface tension and the adhesive force between the liquid and the solid. This phenomenon is called capillarity. As shown in Fig. 2.8, d is the diameter of the tube, θ the contact angle of the liquid to the wall, ρ the density of liquid, and h the mean height of the liquid surface. The following equation is obtained owing to the balance between the adhesive force of liquid stuck to the wall, trying to pull the liquid up the tube by the surface tension, and the weight of liquid in the tube:

$$\pi d T \cos \theta = \frac{\pi d^2}{4} \rho g h$$

or

$$h = \frac{4T \cos \theta}{\rho g d} \quad (2.8)$$

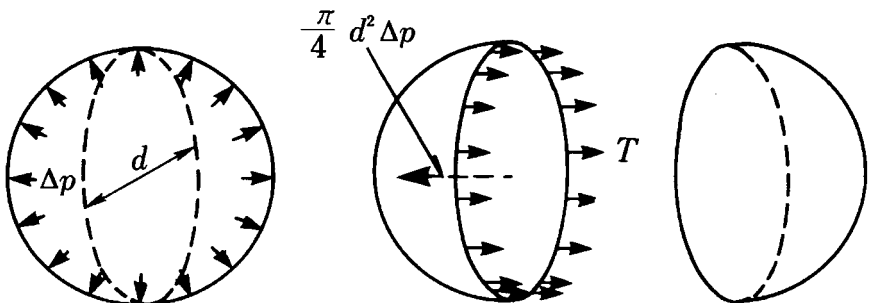


Fig. 2.6 Balance between the pressure increase within a liquid drop and the surface tension

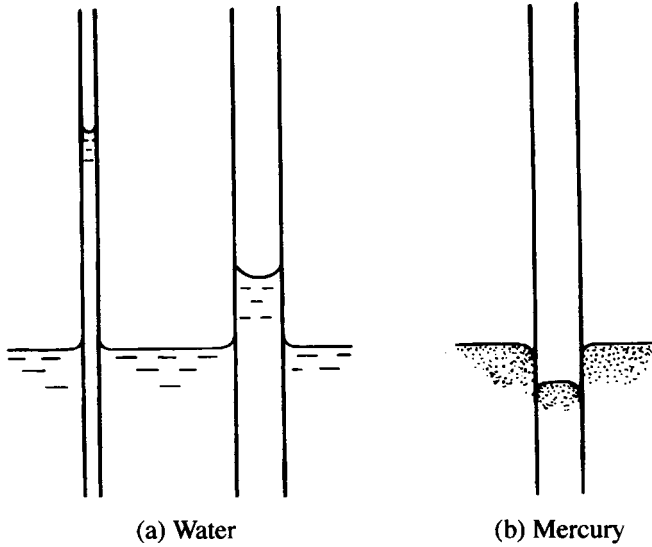


Fig. 2.7 Change of liquid surface due to capillarity

Whenever water or alcohol is in direct contact with a glass tube in air under normal temperature, $\theta \simeq 0$. In the case of mercury, $\theta = 130^\circ - 150^\circ$. In the case where a glass tube is placed in liquid,

$$\left. \begin{array}{l} \text{for water} \quad h = 30/d \\ \text{for alcohol} \quad h = 11.6/d \\ \text{for mercury} \quad h = -10/d \end{array} \right\} \quad (2.9)$$

(in mm). Whenever pressure is measured using a liquid column, it is necessary to pay attention to the capillarity correction.

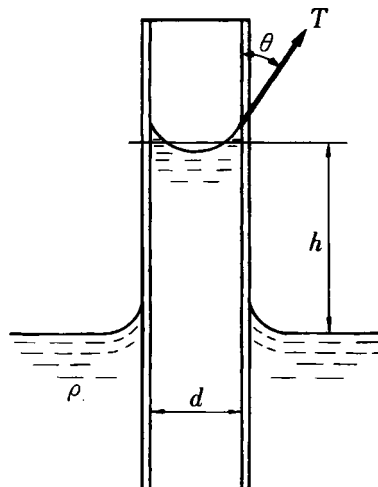


Fig. 2.8 Capillarity

2.6 Compressibility

As shown in Fig. 2.9, assume that fluid of volume V at pressure p decreased its volume by ΔV due to the further increase in pressure by Δp . In this case, since the cubic dilatation of the fluid is $\Delta V/V$, the bulk modulus K is expressed by the following equation:

$$K = \frac{\Delta p}{\Delta V/V} = -V \frac{dp}{dV} \quad (2.10)$$

Its reciprocal β

$$\beta = 1/K \quad (2.11)$$

is called the compressibility, whose value directly indicates how compressible the fluid is. For water of normal temperature/pressure $K = 2.06 \times 10^9$ Pa, and for air $K = 1.4 \times 10^5$ Pa assuming adiabatic change. In the case of water, $\beta = 4.85 \times 10^{-10}$ 1/Pa, and shrinks only by approximately 0.005% even if the atmospheric pressure is increased by 1 atm.

Putting ρ as the fluid density and M as the mass, since $\rho V = M = \text{constant}$, assume an increase in density $\Delta \rho$ whenever the volume has decreased by ΔV , and

$$K = \rho \frac{\Delta p}{\Delta \rho} = \rho \frac{dp}{d\rho} \quad (2.12)$$

The bulk modulus K is closely related to the velocity a of a pressure wave propagating in a liquid, which is given by the following equation (see Section 13.2):

$$a = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \quad (2.13)$$

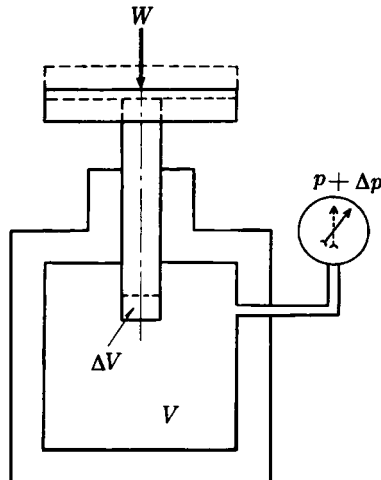


Fig. 2.9 Measuring of bulk modulus of fluid

Table 2.5 Gas constant R and ratio of specific heat κ

Gas	Symbol	Density (kg/m^3) (0°C , 760 mm Hg)	R (SI) $\text{m}^2/(\text{s}^2 \text{K})$	$\kappa = c_p/c_v$
Helium	He	0.178 5	2 078.1	1.66
Air	–	1.293	287.1	1.402
Carbon monoxide	CO	1.250	296.9	1.400
Oxygen	O ₂	0.089 9	4 124.8	1.409
Hydrogen	H ₂	1.429	259.8	1.399
Carbon dioxide	CO ₂	1.977	189.0	1.301
Methane	CH ₄	0.717	518.7	1.319

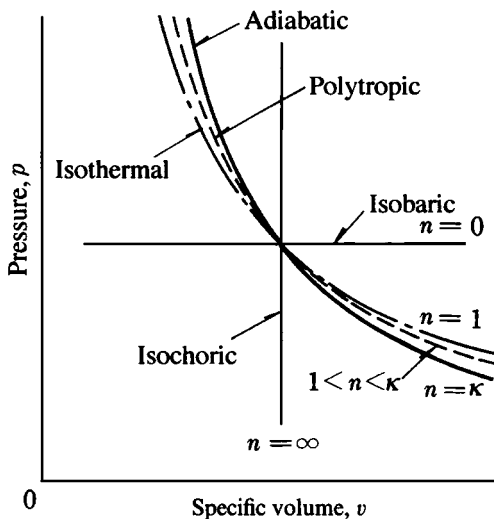
2.7 Characteristics of a perfect gas

Let p be the pressure of a gas, v the specific volume, T the absolute temperature and R the gas constant. Then the following equation results from Boyle's–Charles' law:

$$pv = RT \quad (2.14)$$

This equation is called the equation of state of the gas, and $v = 1/\rho$ (SI) as shown in eqn (2.3). The value and unit of R varies as given in Table 2.5.

A gas subject to eqn (2.14) is called a perfect gas or an ideal gas. Strictly speaking, all real gases are not perfect gases. However, any gas at a considerably higher temperature than its liquefied temperature may be regarded as approximating to a perfect gas.


Fig. 2.10 State change of perfect gas

The change in state of a perfect gas is expressed by the following equation:

$$pv^n = \text{constant}$$

where n is called the polytropic exponent. As this value changes from 0 to ∞ , as shown in Fig. 2.10, the state of gas makes five kinds of changes known as isobaric, isothermal, polytropic, adiabatic and isochoric changes. In particular, in the case of adiabatic change, $n = \kappa$ is obtained. Here κ is the ratio of specific heat at constant pressure c_p to specific heat at constant volume c_v , called the ratio of specific heats (isentropic index). Its value for various gases is given in Table 2.5.

2.8 Problems

1. Derive the SI unit of force from base units.
2. Express the viscosity and the kinematic viscosity in SI units.
3. The density of water at 4°C and 1 atm is 1000 kg/m^3 . Obtain the specific volume v under such conditions.
4. Obtain the pressure in SI (Pa) necessary for shrinking the volume of water by 1% at normal temperature and pressure. Assume the compressibility of water $\beta = 4.85 \times 10^{-10} \text{ 1/Pa}$.
5. When two plates are placed vertically on liquid as shown in Fig. 2.11, derive the equation showing the increased height of the liquid surface between the plates due to capillarity. Also when flat plates of glass are used with a 1 mm gap, what is the increased height of the water surface?

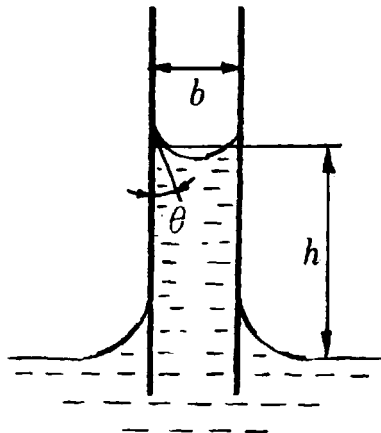


Fig. 2.11

6. Water at 20°C contains a bubble of diameter 1 mm. How much higher is the internal pressure of this bubble compared with the outside pressure?
7. How much force is necessary to lift a ring, diameter 20 mm, made of fine wire, and placed on the surface of water at 20°C ?
8. As shown in Fig. 2.12, a cylinder of diameter 122 mm and length 200 mm is placed inside a concentric long pipe of diameter 125 mm. An oil film is introduced in the gap between the pipe and the cylinder. What force is necessary to move the cylinder at a velocity of 1 m/s? Assume that the dynamic viscosity of oil is 30 cSt and the specific gravity is 0.9.

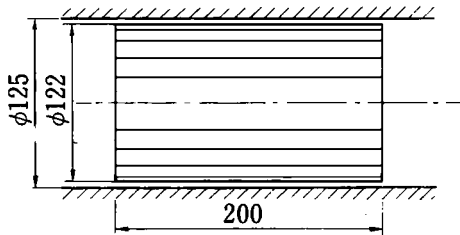


Fig. 2.12

9. Calculate the velocity of sound propagating in water at 20°C . Assume that the bulk modulus of water $K = 2.2 \times 10^9$ Pa.